

NUCLEONS IN THE COVARIANT QUARK–DIQUARK MODEL

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Abstract

We introduce diquarks as separable correlations in the two-quark Green's function to facilitate the description of baryons as relativistic three-quark bound states. These states then emerge as solutions of Bethe–Salpeter equations for quarks and diquarks that interact via quark exchange. Approximating quark and diquark propagators by the corresponding free ones, we calculate nucleon static properties and form factors. For the description of production processes off the nucleon, we consider various dressing functions for the propagators to remove unphysical thresholds. Results for kaon photoproduction, $\gamma p \rightarrow K\Lambda$, and associated strangeness production, $pp \rightarrow pK\Lambda$, allow us to draw conclusions on the permissibility of different dressing functions.

1 The Covariant Quark–Diquark Model

In the last few years enormous experimental progress in studying hadronic properties in the few GeV regime has been made that encourages the study of relativistic, explicitly covariant models of baryons. We adopt a Green's function approach and study the 3-quark correlation function whose poles signal the appearance of bound states, the baryons. To this end, the propagation of a single quark and of two quarks need to be investigated. For single quarks, it has been confirmed by Dyson–Schwinger studies [1, 2] that they acquire a dynamically generated constituent mass by gluon dressing. Additionally these studies indicate that the quark propagator does not possess poles for real values of its momentum squared and is thus confined. Therefore we parameterize the single quark propagator (in Euclidean space) by

$$S^{(k)}(p) = \frac{i\not{p} - m_q}{p^2 + m_q^2} f_k\left(\frac{p^2}{m_q^2}\right), \quad (1)$$

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Quarks have a constituent mass $m_q \approx 400$ MeV. We remark that for setting the dressing function $f_0 = 1$, eq. (1) describes a free quark. Absence of real poles can be achieved by the choices [3]

$$\begin{aligned} f_1(x) &= \frac{1}{2} \left\{ \frac{x+1}{x+1-i/d} + \frac{x+1}{x+1+i/d} \right\} , \\ f_2(x) &= 1 - \exp[-d(1+x)] , \quad f_3(x) = \tanh[d(1+x)(1+x^*)] . \end{aligned} \quad (2)$$

For the first choice, the free mass pole is shifted to a pair of complex conjugate poles thereby mimicking creation and decay processes of quarks that cancel each other in physical processes. The second choice models the propagator as an entire analytical function, at the expense of an essential singularity at timelike infinity, $p^2 \rightarrow -\infty$. Applications of this form may be found in ref. [1]. The third form prescribes a pole free form with an asymptotic free particle behavior in all directions of the complex momentum plane. This can be achieved only by adopting a non-analytic dressing function.

Equipped with models for the single quark propagator we investigate the relativistic 3-quark problem. We neglect any three-particle irreducible interaction graphs between the quarks, which defines the well-known Faddeev problem. For the 2-quark correlations, we observe that any two quarks within baryons have to be in a color $\mathbf{\bar{3}}^c$ representation and that the gluon exchange between quarks in the representation $\mathbf{\bar{3}}^c$ is attractive. Furthermore lattice results indicate strong diquark correlations in the scalar channel [4]. This motivates a separable *ansatz* for the quark-quark t -matrix of the form

$$t_{\alpha\gamma,\beta\delta}^{\text{sep}}(p, q, P) = \chi_{\gamma\alpha}(p) D(P) \bar{\chi}_{\beta\delta}(q) + \chi_{\gamma\alpha}^\mu(p) D^{\mu\nu}(P) \bar{\chi}_{\beta\delta}^\nu(q) . \quad (3)$$

Here, P is the total momentum of the incoming and the outgoing quark-quark pair, p and q are the relative momenta between the quarks. $\chi_{\alpha\beta}(p)$ and $\chi_{\alpha\beta}^\mu(p)$ are vertex functions of quarks with a scalar and an axialvector diquark, respectively. We parameterize the finite size of the diquark vertices by a dipole form. We take the associated width parameter, that directly influences the proton electric radius, to be of the order 300–400 MeV. The inclusion of axialvector diquarks is the minimal requirement to describe decuplet baryons and, as it turns out later, is crucial for describing the nucleon electromagnetic form factors correctly. For simplicity, the diquark propagators $D^{[\mu\nu]}$ are taken to be free propagators of a spin-0 [spin-1] particle multiplied by the dressing functions defined in eq. (2).

Having imposed the separable *ansatz* (3) the Faddeev equations reduce to a coupled system of Bethe–Salpeter equations describing baryons as bound states

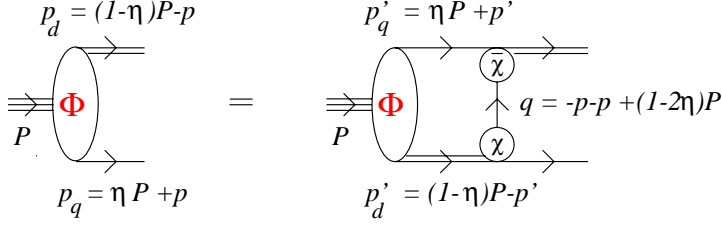


Figure 1: The baryon Bethe-Salpeter equation. The momentum partitioning parameter η distributes the relative momentum p (p') over quark and diquark.

of quarks and diquarks which interact by quark exchange. This interaction is by virtue of the color degree of freedom attractive and restores the Pauli principle. For the nucleon these equations reads

$$\int \frac{d^4 p'}{(2\pi)^4} K(p, p', P) \begin{pmatrix} \Psi^5 \\ \Psi^{\mu'} \end{pmatrix} (p', P) = 0 \quad (4)$$

The interaction part of the kernel K ,

$$K(p, p', P) = (2\pi)^4 \delta(p - p') S^{-1}(p_q) \begin{pmatrix} D^{-1} & 0 \\ 0 & (D^{-1})^{\mu'\mu} \end{pmatrix} (p_d) + \frac{1}{2} \begin{pmatrix} \chi S^T(q) \bar{\chi} & -\sqrt{3} \chi^{\mu'} S^T(q) \bar{\chi} \\ -\sqrt{3} \chi S^T(q) \bar{\chi}^{\mu} & -\chi^{\mu'} S^T(q) \bar{\chi}^{\mu} \end{pmatrix}, \quad (5)$$

is given by the quark exchange (for the definition of the involved momenta see Fig. 1). We have solved these equations without further reduction and thus obtained covariant spinorial wave functions $\Psi^{[\mu]}$.

In a study employing free quark and diquark propagators [5] we calculated the nucleon electromagnetic form factors. Gauge invariance and correct charge and norm were guaranteed by coupling the photon to all possible places in the kernel of the Bethe-Salpeter equation given in eq. (5). The results for the electric form factors (up to momentum transfers of 2.5 GeV^2) are in good agreement with the experimental data, nevertheless it turned out to be impossible to obtain a simultaneous correct description of the nucleon magnetic moments and the mass of the Δ isobar. Due to the free particle thresholds $m_q > 411 \text{ MeV}$ had to be chosen to obtain a bound Δ and these constituent quark masses yielded proton magnetic moments $\mu_p \approx 1.9$. For lower masses $m_q = 360 \text{ MeV}$ we found $\mu_p \approx 2.5$, thus illustrating the necessity to avoid the free-particle poles for the quarks. We found that 20–25 % axialvector correlations (measured by the ratio of the norm contributions stemming from Ψ and Ψ^μ) are needed to describe the ratio of proton electric to magnetic form factor in accordance with experiment.

p_K
 $+$
 P_Λ

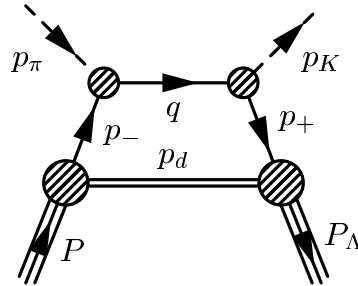


Figure 2: *Left panel:* Main contribution to $p\gamma \rightarrow K\Lambda$, *right panel:* Handbag diagram contributing to the reaction $pp \rightarrow pK\Lambda$ as a subprocess. The incoming pion couples to the ‘spectating’ proton.

2 Production Processes and Quark Confinement

In ref. [3] we studied the processes $\gamma p \rightarrow K\Lambda$ and $pp \rightarrow pK\Lambda$ within the covariant diquark model in impulse approximation. These processes have been investigated experimentally [6, 7]. Assuming for the moment just scalar correlations to be present in p and Λ , the impulse approximation for both processes is restricted to diquark spectator graphs of which the handbag-type ones are shown in Fig. 2. A kinematical analysis shows that the momentum q of the intermediate quark is far in the timelike domain, thus, using free quarks and diquarks, amplitudes corresponding to the handbag diagrams would show unphysical thresholds. For this reason the dressing functions of eq. (2) have been introduced and the quality of their parameterization of the timelike domain can be assessed by studying the above mentioned production processes.

We solved the Bethe–Salpeter equations for nucleon, Λ and the other octet baryons using the dressed propagators. For the non-analytic form (f_3) we found a violation of relativistic translation invariance. We calculated the electromagnetic form factors to further constrain model parameters. The formalism shortly described in the previous section had to be adapted to describe gauge-invariantly the photon couplings to quarks and diquarks. For the case of non-analytic propagators we found that the well-known Ball–Chiu construction (see *e.g.* refs. [1, 2]) had to be modified in a way that depends on the frame that is used to calculate the form factors. Therefore already at this level it is safe to conclude that non-analytic propagators can be excluded in the search for an effective confinement parameterization due to the problems with relativistic invariance.

Dressing the quark–photon vertices leads to an enhancement of μ_p , being

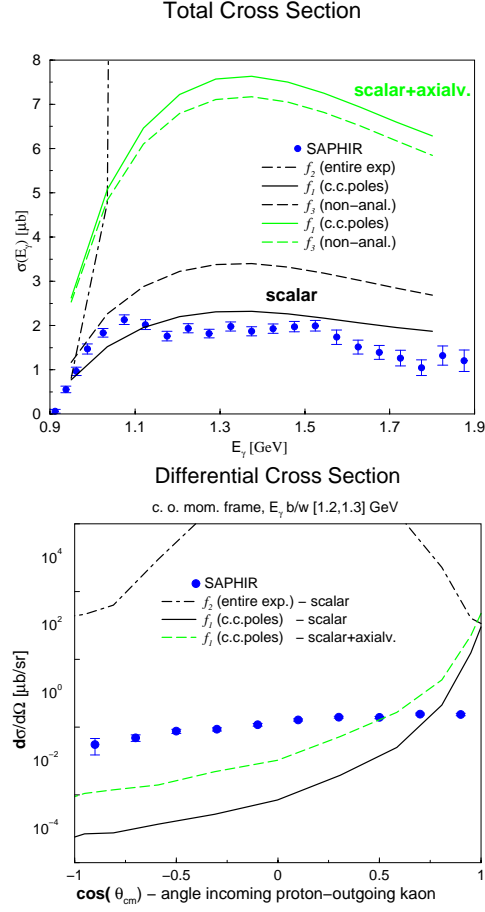


Figure 3: The process $\gamma p \rightarrow K\Lambda$. *Left panel:* Total cross section *vs.* the energy of the incoming photon. The black curves show results for data sets with scalar diquark correlations only but with different propagator dressing functions. The light curves show results when axialvector correlations are included using the dressing functions f_1 and f_3 . *Right panel:* Differential cross section in the center-of-momentum frame.

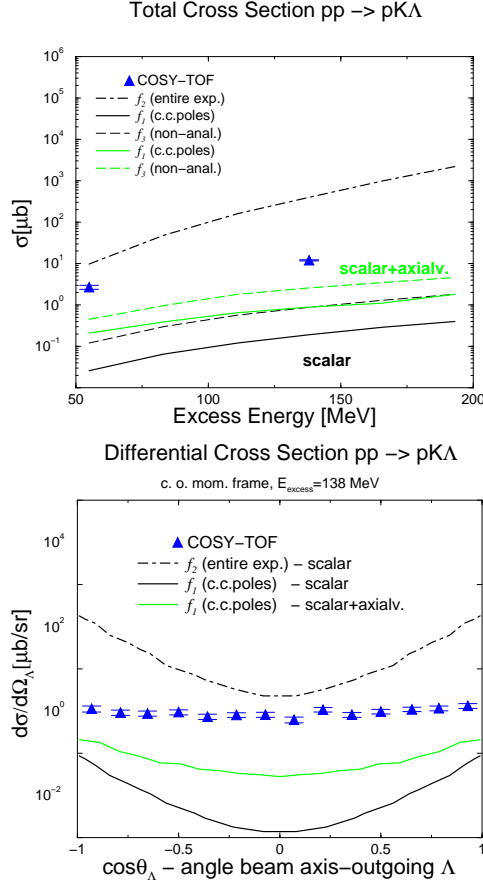


Figure 4: The process $pp \rightarrow pK\Lambda$. *Left panel:* Total cross section *vs.* the excess energy. *Right panel:* Differential cross section in the center-of-momentum frame at an excess energy of 138 MeV.

close to the experimental value for all choices of the dressing function. The inclusion of 20 % axialvector correlations in the nucleon yields considerable improvements on the ratios μ_p/μ_n , G_E/G_M and the values of the Σ and Ξ hyperon masses. Therefore we have extended the study of the production processes to impulse approximation diagrams with the quark being spectator. These arise for a non-zero axialvector admixture in the baryon wave function. The results for kaon photoproduction are shown in Fig. 3. Here the most striking observation is the drastic increase of the cross section above threshold for the choice of the entire analytical function f_2 . The latter is enhanced in the timelike region and thus the handbag diagram of figure 2 dominates, yielding completely unphysical results. For the other two choices, f_1 and f_3 ,

the handbag diagram yields negligible cross sections compared to a t -channel K exchange diagram. The overestimation of the cross section for the data sets using scalar and axialvector correlations is due to an oversimplified treatment of the kaon-quark vertex [3]. The results for the process $pp \rightarrow pK\Lambda$, given in Fig. 4, show similar, unphysical enhancement of the cross section for the choice f_2 for the dressing function. For the other choices, total cross sections are too small and differential cross sections show a dip which is not seen in experiment. This indicates shortcomings of the impulse approximation, however, as for the form factors, the inclusion of axialvector correlations in the baryons yields considerable improvement.

In summary, a picture of the nucleon including scalar and axialvector di-quark correlations together with an effective parameterization of confinement in form of dressing functions with complex conjugate poles seems to be worth further study.

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